Hadron spectroscopy based on relativistic Schrödinger - like wave equations

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Abstract

Relativistic potential-type equations are proposed which approximately reproduce important features and constraints of field-theoretical models and can be a useful tool in hadron spectroscopy. Within this approach, the Regge-trajectory parameters and some properties of higher radial excitations in the light quark sector are discussed. .

1 Introduction

The phenomena in the long-distance (or the low-momentum) domain of hadronic interactions are known to be dominated by non-perturbative mechanisms of QCD. While the lattice QCD simulations are widely accepted to provide the most direct description of these interactions, they contain the inherent difficulties connected with a finite lattice size and the violation of translational and rotational invariance. Therefore, those non-perturbative methods which maintain the mentioned symmetries are also needed, and the potential model either in the nonrelativistic or relativistic wave equation context belong to this category. Furthermore, the continued accumulation of data on availability of hadrons with the same quantum numbers, *e.g.*, the vector mesons produced in the e^+e--annihilation, τ-lepton decays, photo- and electro-production reactions as well as continued and more elaborated treatments of data [1],[2] and phenomenological analyses of hadron form factors [3] seem to favour the use of more sophisticated mixing schemes of constituent configurations in the considered resonance states. So, the mixing of quarkonia ($q\bar{q}$), the multiquark, *e.g.* ($q^2\bar{q}^2$), and hybrid ($q\bar{q}g$)-states is observed as a really important problem of hadron spectroscopy. We believe that in solving this problem, those models will be advantageous which deal with translation-invariant wave functions depending on a correct number of degrees of freedom in the corresponding configuration space of two (or more) particles. It seems natural to expect that the use of the same approach based on 2-, 3-, or 4-body relativistic equations with effective potential interactions between the constituents can present a sufficiently consistent approximate scheme to consider the indicated problem before resorting to more reliable but much more sophisticated numerical approaches to the genuine nonperturbative lattice QCD. In this paper, we outline some features of the approach, where we are going to keep as close as possible to known methods of dealing with one-particle relativistic and few-body nonrelativistic wave equations while discussing the spectroscopy of hadrons on the basis of Schrödinger-like relativized wave equations for bound quark-gluon systems.

2 The orbital and radial trajectories of light mesons

The valence quark model is known to be very successful in the description of mesons and baryons as $q\bar{q}$- and q^3-states, where quarks interact via effective potentials [4]. The principally important ingredient of these interactions is the linearly rising potential of confinement. This result follows from the "quenched" approximation of the lattice QCD giving the flavour-independent force between static colour sources combined in the colour singlet states. The "unquenching" procedure, *i.e.*, inclusion of vacuum polarization due to the light quark loops can result both in the system- and state - dependence of effective potentials and in the modification or even termination of the linear behaviour of the confinement potential. A relevant way to locate these effects is to study the properties of particle states with the highest orbital and radial excitation of hadrons.

In this section, we report some results on mass spectra and radial structure parameters of higher excitations of quarkonia following the formulation proposed earlier [5] for the potential approach to relativistic bound quark systems. A specific feature of the formulated approach is that the equations constructed include the squared forms of the Dirac hamiltonians of each particle interacting with all other particles of a system and additional variation conditions that define the one-particle energies via the total energy of a system, the only spectral parameter entering into the wave equation.

Here we concentrate on the topic of Regge–trajectories for light quarkonia to emphasize the special role of squared forms of long-range potentials of confinement in evaluation of characteristics of such states. It is just the squared form of the linear world-scalar potential of confinement that provides linearity of the Regge trajectories with the slope which can be made close to that following from the relativistic string theory. To illustrate this, we write first a general structure of our relativistic two-body equation in the form

$$\frac{1}{2W}(W^2+\vec{P}^2+M^2(p^2,r^2,..))\Psi(1,2)=[\sum_{i=1,2}\frac{1}{2\varepsilon_i}(\varepsilon_i^2+\vec{p}_i^2+m_i^2)+\hat{V}]\Psi(1,2)=W\Psi(1,2). \quad (1)$$

$$\vec{P}=\vec{p}_1+\vec{p}_2, W=\varepsilon_1+\varepsilon_2.$$

The solution of the eigenvalue problem, *i.e.* the finding of the eigenmass M for a given mass-operator M^2 in (1), is seen to correspond to zero of the inverse of the one-particle propagator written in the covariant form

$$P_\mu P^\mu - M^2 = 0$$

The interaction kernel V in (1) is defined when in the equation for noninteracting particles in their c.m.s.

$$W_0\Psi_0(1,2)=[\sum_{i=1,2}\frac{1}{2\varepsilon_i}(\varepsilon_i^2+\hat{h}_0^2(i)]\Psi_0(1,2)=[\sum_{i=1,2}\frac{1}{2\varepsilon_i}(\varepsilon_i^2+\vec{p}_i^2+m_i^2)]\Psi_0(1,2) \qquad (2)$$

we alternately replace $\hat{h}_0^2(i)$ by the squared forms of Dirac operator $\hat{h}(i)$ for either particle in the field of another one

$$W\Psi(1,2)=\{[\omega_1(\frac{1}{2\varepsilon_1}(\varepsilon_1^2+\hat{h}^2(1))+\frac{1}{2\varepsilon_2}(\varepsilon_2^2+\hat{h}_0^2(2))]+[1\leftrightarrow 2]\}\Psi(1,2), \qquad (3)$$

$$\hat{h}^2(i) = \vec{p}_i^2 + m_i^2 + 2m_i V_s(r_{ij}) + V_s(r_{ij})^2 + 2\varepsilon_i V_v(r_{ij}) - V_v{}^2(r_{ij}) + \text{spin-dependent terms}, \quad (4)$$

where the weight factors ω_i are normalized to unity:$\omega_1 + \omega_2 = 1$.If $m_1 = m_2$, then, by symmetry arguments, one can expect $\omega_1 = \omega_2 = \frac{1}{2}$. When $m_1 \neq m_2$, we suggest to use the simple relation $\omega_1/\omega_2 = \varepsilon_2/\varepsilon_1$ that reproduces, for the static limit $m_j \to \infty$, the correct form of the corresponding one-particle equation in the field of a fixed center, *e.g.*, the Klein-Gordon-Fock equation for spinless particles. In our case, the long-range confinement "potential" is seen to be, in fact, the mass term parametrically dependent on parton configuration coordinates. As usual, in the spin-independent part of our interaction kernel, we retain a world-scalar part and the 0-th component of the vector interaction potential which survive in nonrelativistic approximation.

To be phenomenologically acceptable, our relativistic equation with a given Lorentz structure of the confining kernel should possess stable physical solutions, *i.e.*, the binding energy should be real, and the state should be localized in a finite region of the coordinate space. As the presumed scalar and vector parts of the confining interaction squared enter into the equation with opposite signs, the scalar part is required to be stronger than the vector confinement potential: $V_v \leq V_s$. The squared Coulomb potential gives the most singular part of the interaction. Hence for typical scales of the ground state and higher radial states of quarkonia with $l = 0$, the effective $\alpha_s(r)$ values should be bounded from above:

$$Im[\sqrt{(l+\frac{1}{2})^2 - \frac{1}{2}(\frac{4}{3}\alpha_s)^2}] = 0, \alpha_s \leq \frac{3}{4\sqrt{2}} \simeq .53. \quad (5)$$

This means that the effective coupling constant of QCD should be taken as "freezing" at the value $\sim .5$ by one of the proposed nonperturbative mechanisms (see, e.g., [6]) in the infrared region.

Below we are going to invoke also some ideas of the string-approximated QCD, or rather, a string-like solution of the bag model [7], to further specify the structure of the confining interaction and then to compare emerging results with the model-independent constraints [8] on general relativistic bound states. It will be shown that the string-like structure of QCD, resulting *e.g.*, from a semiclassical consideration of the rotating and deformed ("fat-string-like") bag [7] , can be represented by a much more simple potential model of mesons that is able to approximately reproduce some important features and results of the "microscopic" field-theoretical models.

The famous constant B of the MIT-bag model is known to define the volume energy of the space region inside an extended hadron, where nonperturbative vacuum fluctuations of coloured fields are at least partially suppressed, and it defines also the balance of the inward and outward pressure of vacuum fields and the coloured fields created by partons of a given hadron. This constant is considered to be connected with the contribution to the total hadron mass of the "abnormal" part (*i.e.*, with the nonzero trace) of the QCD energy-momentum tensor. The general model-independent statement stressed by Ji [8] is that the ratios of contributions to the mass of the "abnormal"($\bar{T}_{\mu\nu}$) and traceless ($\hat{T}_{\mu\nu}$) parts of the energy-momentum tensor are

$$\bar{M}/\hat{M} = 1/3, \quad (6)$$

$$\hat{M}/M_{tot} = 3/4. \quad (7)$$

The effective string-potential, or rather, the energy of the string-like configuration of two static $\bar{q}q$-sources of a colour field, is found, following [7], to be

$$V_s(r) = a_{tot} r = (\bar{a} + \hat{a}) r = (\frac{128}{3} \pi \alpha_s B)^{\frac{1}{2}}, \tag{8}$$

where $\bar{a} = \hat{a}$ represents, respectively, the contribution of the chromoelectric field of quarks ($i.e.$, the traceless part of the gluon energy-momentum tensor), and the bag energy is connected with the gluon condensate, that is the trace-anomaly of QCD.

It was shown in [7] that the classic consideration of rotation of the chromoelectric "flux-tube" leads to a Regge-type relation between the classic *nonquantized* angular momentum J and mass M_{string}

$$J = \frac{1}{2\pi a} M_{string}^2 \tag{9}$$

where two parts of M_{string}, $\hat{M}_{string}$, and $\bar{M}_{string}$ verify the general relations (6) and (7).

We use V_s in the quantum-mechanical (quasi)potential two-body equation treating it alternately as the effective mass of one or another quark that parametrically depends on the interquark distance. According to the given definition, we obtain for massless quarks, with the neglect of spin-dependent terms,

$$(\vec{p}^2 + \frac{1}{2} a^2 r^2 - \frac{1}{4} W^2) \Psi(1,2) = 0 \tag{10}$$

The solution gives a simple dependence of the mass $M = M(l, n_r)$ on the *quantized* orbital (l) and radial (n_r) quantum numbers

$$W^2 \equiv M_{tot}^2 = 4\sqrt{2} a (l + 2n_r + \frac{3}{2}). \tag{11}$$

characteristic of a harmonic oscillator. The harmonic quasipotential is obtained, however, after squaring the linear world–scalar "potential" of confinement. The adopted way of inclusion of the interaction kernel into our relativistic two-body equation resembles the prescription of the so-called "spectator"- type equation [9], where, alternately, one of the particles is put off-mass-shell, while the other is taken to be on-mass-shell.

According to (11), the Regge-trajectory slope $\alpha'(0) = (4\sqrt{2}a)^{-1}$ is within 10% of the value $\alpha'(0) = (2\pi a)^{-1}$ following from the relativistic string theory. Replacing formally a in (10) by $\hat{a} = (1/2)a$, we keep in M^2 only contributions of the traceless $\hat{T}_{\mu\nu}$-part of massless quarks and gluons. Therefore, we have

$$\frac{\hat{M}^2}{M_{tot}^2} = \frac{1}{2} \tag{12}$$

which is again within 10% of the model-independent ratio (7).

The intercept $\alpha_J(0)$ of the leading $J = l + 1$-trajectory is $-.5$ according to (11), hence unrealistic. To calculate the realistic intercepts of Regge-trajectories, one should include the spin-dependent potentials, presumably the spin-orbit interaction induced by a scalar confinement "potential".

In this case, one can eliminate a still unknown parameter, using the value of mass and known quantum numbers of a certain state to evaluate masses of the states with other quantum numbers but with the same quark content. We note also that by analogy with the nonrelativistic case one can calculate the mean value of the commutator of the mass-operator $M^2(p^2, r^2, ..)$ with the scalar product $(\vec{p}\vec{r})$

$$\langle \Psi_{l,n_r} | [M^2(p^2, r^2, ..), (\vec{p}\vec{r})]_- | \Psi_{l,n_r} \rangle = 0 \tag{13}$$

to get the "virial theorem", demonstrating that the contribution of the mean value of the kinetic energy of two massless valence quarks to the value of the total mass of a highly excited meson is equal to the corresponding contribution of the mean value of the "potential energy" that is connected with the energy integrated over gluon degrees of freedom of a given hadron. It is further tempting to interpret this fact as giving a hint for approximate equipartition, on the scale pertinent to a given bound state, of the total momentum of a hadron in the "infinite momentum" frame between the valence quark and gluon-sea partons, the fact, following from the known moment of the nucleon structure function measured on the scales of deep inelastic lepton-hadron scattering.

Further, we discuss further in brief some characteristics of higher radial excitation of light vector quarkonia. This question is especially timely in view of recent development of the Vector Meson Dominance model applications [3] to the analysis of nucleon electromagnetic form factors both in the spacelike and timelike regions of the transferred momenta Q^2. With the approximate equation (10), we obtain "asymptotic" relations between masses of resonances with high spins J, lying on the same trajectory, and masses of higher radial excitations with the same spin

$$m^2(J, n_r) - m^2(J', n_r) = \alpha'(0)^{-1}(J - J') \tag{14}$$

$$m^2(J, n_r) - m^2(J, n_r') = 2\alpha'(0)^{-1}(n_r - n_r') \tag{15}$$

If we take $\rho(2130)$ [10] as one of these higher radial states, then the next two are $\rho(2600)$ and $\rho(3000)$ according to (15) and the value $\alpha'(0) \simeq .9GeV^{-2}$. Curiously enough, a ρ - type resonance with a mass close to 2.6 GeV was suggested in [3] on the basis of analysis of nucleon form factors. The isoscalar partner $\omega(3000)$ of the second state, presumably, degenerated with it, is seen to be near in mass to the J/ψ - resonance, and it can play a role in the enhancement of certain strong decays of J/ψ. This situation deserves a more detailed study.

In the approach constructed, the energy functional has a recognizable quasi-nonrelativistic form, therefore, the perspective is open up to combine the accumulated experience in approximate solutions of the spectral problems in the nonrelativistic (NR) domain with the description of relativistic motion of hadron constituents.To this end, one should to have preferably an analytic, although approximate, solution of the corresponding NR problem. For example, in our case, one can simply estimate the dependence of the "psi-at-zero" values

$$\psi(0) = \frac{m_{red}}{2\pi} < V'(r) > \tag{16}$$

on masses or quantum numbers of corresponding meson states. Making use of the analogy of our equation (10) with the nonrelativistic Schrödinger equation, we obtain approximate scaling relations for the "psi-at-zero" in the case of highly relativistic radially-excited states, hence, for leptonic widths of the corresponding resonances

$$\psi(0)^2 = \frac{a^2}{4\pi} < r >= \frac{am_{V_n}}{\sqrt{6\pi^3}} \tag{17}$$

$$\Gamma_{ee}(V_n) \sim \psi(0)^2/m_{V_n}^2 \sim (n-1/4)^{-\frac{1}{2}} \tag{18}$$

where m_{V_n} is the mass of the radially excited nS-state of the vector resonance; $n = n_r + 1$, the principal quantum number; n_r, the radial quantum number. To get the absolute values of these leptonic widths, one should consistently include the QCD radiative corrections into the amplitude of $\bar{q}q \rightarrow e^+e^-$ transition. The estimation of total widths of higher radial excitations of vector mesons can proceed as follows. With the meson mass of order 2 GeV or larger, many decay channels are open, and one can rely on the quark-hadron duality idea while assuming the dominant role of the initial quark-parton stage of the reaction that defines the width dependence on quantum numbers and mass of the resonance. The total $q\bar{q}$ cross-section in the S-state is assumed to scale as $m_{V_n}^{-2}$, where m_{V_n} is the mass of the V_{n_r}-meson. At the hadronization stage, we adopt the simplest phase-space correction, assuming its form from a presumably main decay mode. For the isovector ρ_n-type resonances, the apparently conspicuous or, at least, important decay channel is the $\rho(.77\ GeV)\pi\pi$-channel. With adopted assumptions, one can get

$$\Gamma_{tot}(V_{n+1}) \simeq \Gamma_{tot}(V_n)\frac{m_{V_{n+1}}f(\alpha_{n+1})}{m_{V_n}f(\alpha_n)} \tag{19}$$

$$f(\alpha_{\rho_n}) = 1 - \alpha^4 + 4\alpha^2 log\alpha \tag{20}$$

where $\alpha_n = m_\rho/m_{\rho_n}$, and the phase-space formula for the decay $\rho_n(m_n) \rightarrow \rho(.77)2\pi$ has been obtained [11] for massless pions. So, with the input $(m = 2.6\ GeV; \Gamma = .6\ GeV)$ for mass and width of the heaviest claimed ρ-type resonance, we get for the next two resonances lying below the open charm threshold: $(mass; width) \rightarrow (2.98; .77)$ and $(3.34; .92)$. The very large and, seemingly, overestimated widths suggest, nevertheless, that hadronic corrections can be important in calculations of masses of high radially-excited states. This problem is still to be considered.

Concerning perspectives of the study of higher excited states within the outlined approach, one can notice that the lattice QCD simulations with the unquenched light quarks seem to be consistent with the linear behaviour of the confinement potential up to the distance of $r \leq 2\ fm$ [12]. With the help of the virial theorem (2.13) one can relate the mean distance $\sqrt{r_{q\bar{q}}^2} \leq 2\ fm$ between the $q\bar{q}$-pair with the corresponding quantum numbers $(l \leq 15; n_r = 0)$, or $(n_r \leq 9; l = 0)$ of a given bound state which are considerably larger than the corresponding values $l \simeq 6 \div 7$ or $n_r \simeq 4 \div 5$ of the known meson resonances. Hence, it seems that many resonances could still be explored theoretically on the basis of relativistic wave equations with the linear confinement term, playing the role of part of effective quark (or gluon) mass, parametrically dependent on the distance between corresponding force centers.

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